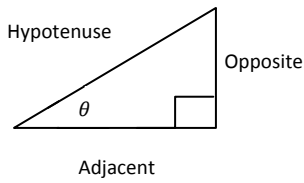# Trigonometry
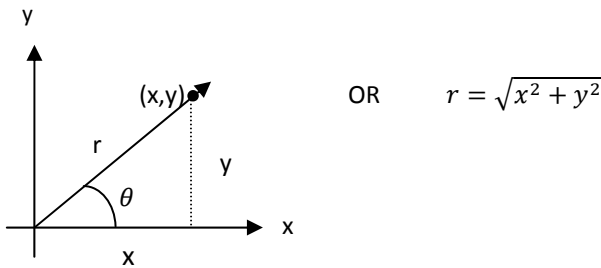
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$

$$\sin \theta = \frac{opp}{hyp} \qquad \csc \theta = \frac{hyp}{opp}$$

$$\cos \theta = \frac{adj}{hyp} \qquad \sec \theta = \frac{hyp}{adj}$$
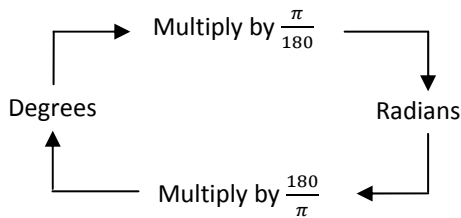
$$\tan \theta = \frac{opp}{adj} \qquad \cot \theta = \frac{adj}{opp}$$

Circular function definitions, where θ is any angle

OR $\qquad r = \sqrt{x^2 + y^2}$

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y} \qquad \sec \theta = \frac{r}{x} \qquad \cot \theta = \frac{x}{y}$$

Converting between Radians and Degrees

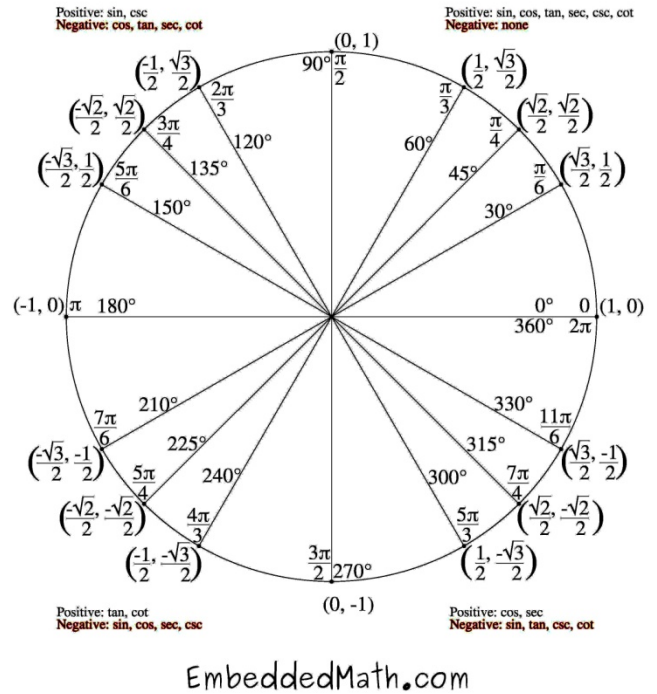Multiply by $\frac{\pi}{180}$

Degrees → Radians

Multiply by $\frac{180}{\pi}$

Helpful Triangles
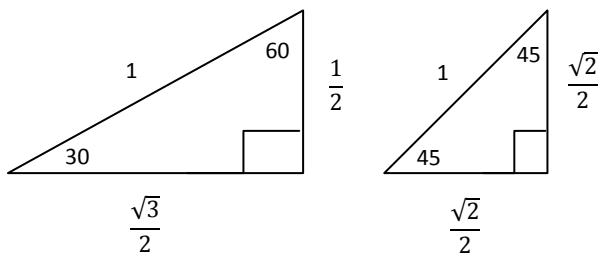
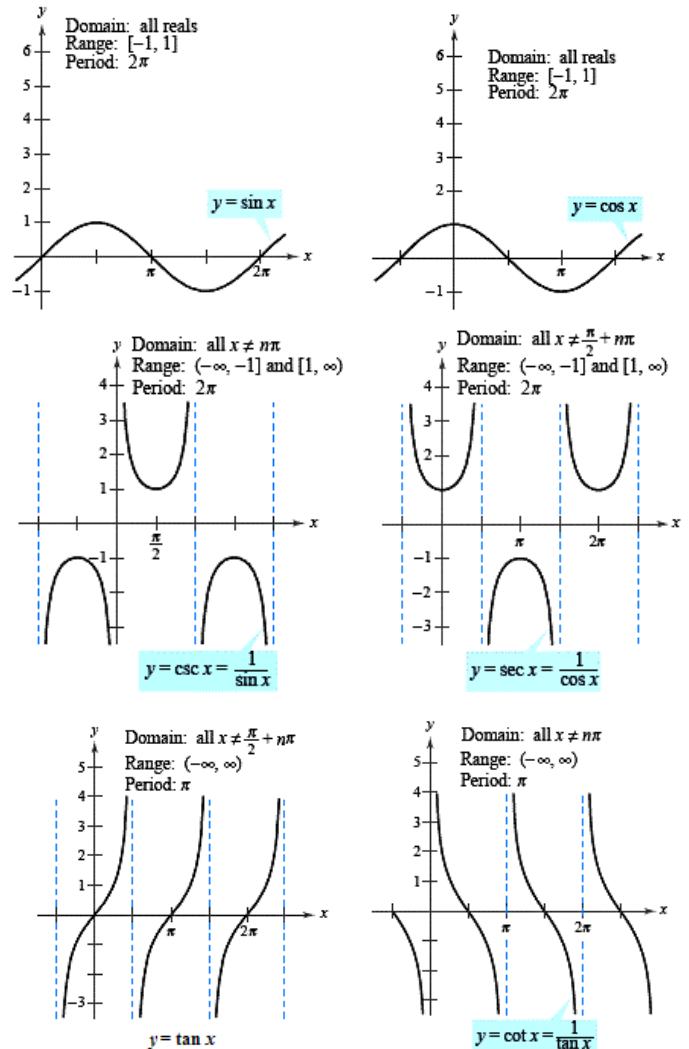
Graphing Sine and Cosine Curves

The graphs of y = A sin (Bx + C) and y = A cos (Bx +C), where B > 0, will have the following characteristics:

Amplitude = |A| $\qquad$ Period $= \frac{2\pi}{B}$

Horizontal Shift $= -\frac{C}{B}$

The Unit Circle

Graphs of Trigonometric Functions

Domain: all reals
Range: $[-1, 1]$
Period: 2π

$y = \sin x$

Domain: all reals
Range: $[-1, 1]$
Period: 2π

$y = \cos x$

Domain: all $x \neq n\pi$
Range: $(-\infty, -1]$ and $[1, \infty)$
Period: 2π

$y = \csc x = \frac{1}{\sin x}$

Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1]$ and $[1, \infty)$
Period: 2π

$y = \sec x = \frac{1}{\cos x}$

Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π

$y = \tan x$

Domain: all $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π

$y = \cot x = \frac{1}{\tan x}$

Trigonometry

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad\qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad\qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

Negative Angle Identities

$$\sin(-x) = -\sin x \qquad\qquad \cos(-x) = \cos x$$

$$\tan(-x) = -\tan x \qquad\qquad \cot(-x) = -\cot x$$

$$\csc(-x) = -\csc x \qquad\qquad \sec(-x) = \sec x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2\cos^2 u - 1 \; = \; 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Half-Angle Formula

$$\sin\frac{u}{2} = \pm\sqrt{\left(\frac{1 - \cos u}{2}\right)}$$

$$\cos\frac{u}{2} = \pm\sqrt{\left(\frac{1 + \cos u}{2}\right)}$$

$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u - v) + \sin(u + v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Law of Sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$