CALCULUS 181

Stewart – Chapter 4

Name: Key

Chapter 4 Test

1. Given $f(x) = 2\cos x + \cos^2 x$ on the interval $(0, 2\pi)$

   a. Find the intervals on which f is increasing and decreasing

$$f'(x) = -2\sin x + 2\cos x(-\sin x)$$
$$= -2\sin x - 2\cos x \sin x$$
$$= -2\sin x(1 + \cos x)$$

$\sin x = 0 \qquad \cos x = -1$

$x = \pi$

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$0 \quad \pi \quad 2\pi$

decreasing $(0, \pi)$

increasing $(\pi, 2\pi)$

   b. Find the coordinant of the absolute minimum and/or maximum. Label which is which

$$f(\pi) = 2(-1) + (-1)^2$$
$$= -2 + 1$$
$$= -1$$

min: $(\pi, -1)$

   c. Find the intervals where the graph is concave up and down

$$f''(x) = (-2\sin x)(1 + \cos x)' + (-2\sin x)'(1 + \cos x)$$
$$= (-2\sin x)(-\sin x) + (-2\cos x)(1 + \cos x)$$
$$= 2\sin^2 x - 2\cos x - 2\cos^2 x$$
$$= 2(1 - \cos^2 x) - 2\cos x - 2\cos^2 x$$
$$= 2 - 2\cos^2 x - 2\cos x - 2\cos^2 x$$
$$= -4\cos^2 x - 2\cos x + 2$$
$$= (-2\cos x + 1)(2\cos x + 2)$$

$\cos x = \frac{1}{2} \qquad \cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$

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$0 \quad \frac{\pi}{3} \quad \pi \quad \frac{5\pi}{3} \quad 2\pi$

d. (continued from previous page) Find the coordinats of the points of inflection

$f\left(\frac{\pi}{3}\right) = 2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = \frac{5}{4}$

$f\left(\frac{5\pi}{3}\right) = 2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = \frac{5}{4}$

POI: $\left(\frac{\pi}{3}, \frac{5}{4}\right), \left(\frac{5\pi}{3}, \frac{5}{4}\right)$

2. Veryify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$f(x) = x^3 - 3x + 2 \qquad [-2,2]$

f is continuous on $[-2,2]$
f is differentiable on $(-2,2)$ > polynomial

So MVT applies.

$\exists c$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$

$$\frac{f(2)-f(-2)}{2-(-2)} = \frac{(8-6+2)-(-8+6+2)}{4} = 1$$

$f'(x) = 3x^2 - 3$

$3c^2 - 3 = 1$

$c^2 = \frac{4}{3}$

$c = \pm \frac{2}{\sqrt{3}}$

3. $\lim_{x\to\infty} \frac{\ln\sqrt{x}}{x^2}$ $\to \frac{\infty}{\infty}$ L'Hospital

$$\lim_{x\to\infty} \frac{\ln\sqrt{x}}{x^2} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}\left(\frac{1}{2}x^{-1/2}\right)}{2x}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{2x}}{2x}$$

$$= \lim_{x\to\infty} \frac{1}{4x^2} \to 0$$

4. $\lim_{x\to 0} \sin 5x \csc 3x$ $= \lim_{x\to 0} \dfrac{\sin 5x}{\sin 3x} \to \dfrac{0}{0}$ L'Hospital

$$\overset{H}{=} \lim_{x\to 0} \frac{5\cos 5x}{3\cos 3x} \to \frac{5}{3}$$

5. $\lim_{x\to\infty} x^{e^{-x}}$

$$y = x^{e^{-x}}$$

$$\ln y = e^{-x}\ln x$$

$$\ln y = \frac{\ln x}{e^x}$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln x}{e^x} \to \frac{\infty}{\infty} \quad \text{L'Hospital}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{1/x}{e^x} \to 0$$

$$y = e^{\ln y}, \text{ so } \lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y}$$

$$= e^0 = 1$$

6. A cylindrical can is to be made to hold 1000 cm^3 of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

$$V = \pi r^2 h$$

$$1000 = \pi r^2 h$$

$$\frac{1000}{\pi r^2} = h$$

$$A = 2\pi r h + 2\pi r^2$$

$$A = 2\pi r\left(\frac{1000}{\pi r^2}\right) + 2\pi r^2$$

$$A = \frac{2000}{r} + 2\pi r^2$$

$$\frac{dA}{dr} = -\frac{2000}{r^2} + 4\pi r$$

$$0 = -\frac{2000}{r^2} + 4\pi r$$

$$\frac{2000}{r^2} = 4\pi r$$

$$4\pi r^3 = 2000$$

$$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.42$$

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5.42

min

$$h = \frac{1000}{\pi(5.42)^2}$$

$$h = 58.74$$

7. Find the coordinates for the points of inflection of the graph $f(x) = x^4 - 4x^3$

$$f'(x) = 4x^3 - 12x^2$$

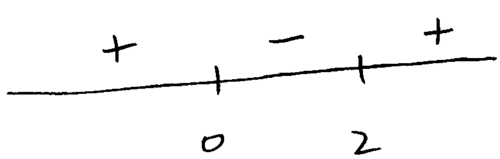
$$f''(x) = 12x^2 - 24x$$

$$0 = 12x^2 - 24x$$

$$0 = x^2 - 2x$$

$$0 = x(x-2)$$

$$x = 0, 2$$



$f(0) = 0$

$f(2) = -16$

POI: $(0,0)$

$(2,-16)$