Square Roots and Other Radicals

Number a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square a^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Cube a^3	1	8	27	64	125	216	343								

Simplifying Radicals

Examples		Rule
$\sqrt{3^2} = 3$ $\sqrt[3]{5^3} = 5$ $\sqrt[4]{7^4} = 7$ $\sqrt[2]{(-2)^2} = \|-2\| = 2$ $\sqrt[3]{(-3)^3} = -3$	⟶	$\sqrt[n]{a^n} = a$ If *n* is odd $\sqrt[n]{a^n} = \|a\|$ If *n* is even
$\sqrt{6} = 6^{\frac{1}{2}}$ $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$ $\sqrt[4]{8^5} = 8^{\frac{5}{4}}$	⟶	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$
$\sqrt{6} \cdot \sqrt{3} = \sqrt{18}$ $\sqrt[4]{5} \cdot \sqrt[4]{4} = \sqrt[4]{20}$	⟶	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$ $\sqrt[3]{\frac{32}{5}} = \frac{\sqrt[3]{32}}{\sqrt[3]{5}} = \frac{\sqrt[3]{8 \cdot 4}}{\sqrt[3]{5}} = \frac{2\sqrt[3]{4}}{\sqrt[3]{5}}$	⟶	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
$4\sqrt{3} + 7\sqrt{3} = (4+7)\sqrt{3} = 11\sqrt{3}$ $6\sqrt{8} + 3\sqrt{2} = 6\sqrt{4 \cdot 2} + 3\sqrt{2} = (6 \cdot 2)\sqrt{2} + 3\sqrt{2}$ $= (12+3)\sqrt{2} = 15\sqrt{2}$ $2\sqrt[3]{7} + 5\sqrt[3]{7} - \sqrt[3]{7} = (2+5-1)\sqrt[3]{7} = 6\sqrt[3]{7}$	⟶	$a\sqrt[n]{c} + b\sqrt[n]{c} = (a+b)\sqrt[n]{c}$

Hint: Look for factors of the radicand (the number under the radical sign) that are perfect squares, cubes, etc.

Rationalizing the Denominator

Simple Rationalization

Examples		Rule
$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{(\sqrt{5})^2} = \frac{3\sqrt{5}}{5}$ $\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$	$\longrightarrow$	$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{\sqrt{b^2}} = \frac{a\sqrt{b}}{b}$
$\frac{5}{\sqrt[3]{3}} = \frac{5}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^{3-1}}}{\sqrt[3]{3^{3-1}}} = \frac{5\sqrt[3]{3^2}}{\sqrt[3]{3^3}} = \frac{5\sqrt[3]{9}}{3}$ $\frac{7}{\sqrt[5]{8}} = \frac{7}{\sqrt[5]{2^3}} \cdot \frac{\sqrt[5]{2^2}}{\sqrt[5]{2^2}} = \frac{7\sqrt[5]{2^2}}{\sqrt[5]{2^5}} = \frac{7\sqrt[5]{4}}{2}$	$\longrightarrow$	$\frac{a}{\sqrt[n]{b^m}} = \frac{a}{\sqrt[n]{b^m}} \cdot \frac{\sqrt[n]{b^{n-m}}}{\sqrt[n]{b^{n-m}}} = \frac{a\sqrt[n]{b^{n-m}}}{\sqrt[n]{b^n}} = \frac{a\sqrt[n]{b^{n-m}}}{b}$

Rationalization using Conjugates

$b + \sqrt{c}$ and $b - \sqrt{c}$ are *conjugates* of each other

Examples		Rule
$\frac{6}{3-\sqrt{2}} = \frac{6}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{6(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{18+6\sqrt{2}}{9-2} = \frac{18+6\sqrt{2}}{7}$	$\longrightarrow$	$\frac{a}{b+\sqrt{c}} = \frac{a}{b+\sqrt{c}} \cdot \frac{b-\sqrt{c}}{b-\sqrt{c}} = \frac{a(b-\sqrt{c})}{(b+\sqrt{c})(b-\sqrt{c})} = \frac{ab-a\sqrt{c}}{b^2-c}$
$\frac{8}{\sqrt{5}+\sqrt{2}} = \frac{8}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{8(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{8\sqrt{5}-8\sqrt{2}}{5-2} =$ $\frac{8\sqrt{5}-8\sqrt{2}}{3}$	$\longrightarrow$	$\frac{a}{\sqrt{b}-\sqrt{c}} = \frac{a}{\sqrt{b}-\sqrt{c}} \cdot \frac{\sqrt{b}+\sqrt{c}}{\sqrt{b}+\sqrt{c}} = \frac{a(\sqrt{b}+\sqrt{c})}{(\sqrt{b}-\sqrt{c})(\sqrt{b}+\sqrt{c})} = \frac{a\sqrt{b}+a\sqrt{c}}{b-c}$