Math 184 Exam 2 SHOW ALL WORK Name key

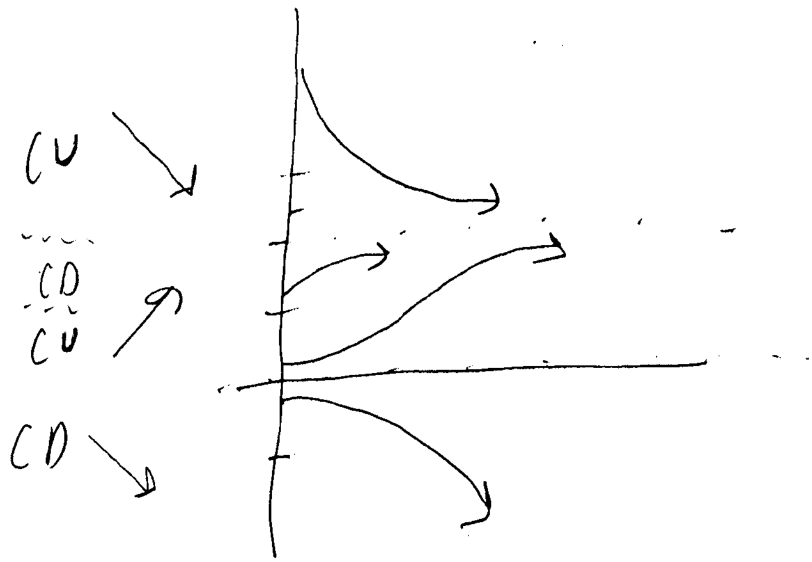
1. Find any equilibrium solutions, use calculus to find the y-intervals for which the solutions are increasing/decreasing and concave up/down, then sketch a phase portrait.

$y' = -y^2 + 2y$

$y' = y(2-y)$

$y = 0, y = 2$

are equi... solutions



If $y<0$, $y'<0$

if $0<y<2$, $y'>0$

if $y>2$, $y'<0$

$y'' = (-2y+2)y'$

possible IP at $y=1$

if $y>2$, $y''>0$

if $y<0$, $y''<0$

if $0<y<1$, $y''>0$

if $1<y<2$, $y''<0$

2. Solve using separation of variables: $\frac{dy}{dx} = x^3y + \sin(x)\cdot y$

$\frac{1}{y}dy = (x^3 + \sin x)dx$

$\ln|y| = \frac{1}{4}x^3 - \cos x + c$

$\boxed{y = Ae^{\frac{1}{4}x^3 - \cos x}, \quad A = \pm e^c}$

9	29
8	
7	25 1
6	55 2
5	5 87
4	9
3	0
2	
1	7

3. Solve using an integrating factor: $y' + x^5 y = x^5$

① $\mu = e^{\int x^5 dx} = e^{\frac{1}{6}x^6}$

② $\frac{d}{dx}\left[e^{\frac{1}{6}x^6} y\right] = e^{\frac{1}{6}x^6} \cdot x^5$

$e^{\frac{1}{6}x^6} y = \int e^{\frac{1}{6}x^6} x^5 dx = e^{\frac{1}{6}x^6} + C$

$\boxed{y = 1 + Ce^{-\frac{1}{6}x^6}}$

4. Solve: $(e^x \cos(y) - \ln(y))dx + (2y - e^x \sin(y) - \frac{x}{y})dy = 0$

Exact? $M_y = -e^x \sin y - \frac{1}{y} = N_x = -e^x \sin y - \frac{1}{y}$ so yes

① $F_x = M$: $F = \int [e^x \cos y - \ln y] dx = e^x \cos y - x \ln y + h(y)$

② $F_y = N$: $\cancel{-e^x \sin y} - \cancel{\frac{x}{y}} + h'(y) = 2y - \cancel{e^x \sin y} - \cancel{\frac{x}{y}} \to h(y) = y^2$

so $F(x,y) = C$ yields!

$\boxed{e^x \cos y - x \ln y + y^2 = C}$

5. Prove that the functions $\{x^2, x, 1\}$ are linearly independent.

expand column

$$\begin{vmatrix} x^2 & x & 1 \\ 2x & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} = 1 \begin{vmatrix} 2x & 1 \\ 2 & 0 \end{vmatrix} + 0 \left| \quad \right| + 0 \left| \quad \right|$$

$= -2 \neq 0$

so Functions are L.I. on $(-\infty, \infty)$

Let A = amount of money

6. You put \$10000 in a bank account that pays 3% annual interest compounded continuously. Each year you withdraw money at a continuous rate of \$1000 per year. Write an IVP (i.e. include initial conditions) that models the amount of money in the account at time t. DO NOT SOLVE.

$$\boxed{\frac{dA}{dt} = 0.03A - 1000, \quad A(0) = 10000}$$

7. Solve: $y'' = (y')^2$

$v = y' \quad v' = y''$

$\frac{dv}{dx} = v' = v^2$

$\int v^{-2} dv = \int dx$

$-v^{-1} = x + c_1$

$v = -\frac{1}{x+c_1}$

$v = y' = -\frac{1}{x+c_1}$

$y = \int -\frac{1}{x+c_1} dx$

$$\boxed{y = -\ln|x+c_1| + c_2}$$

8. Solve: $y''' + 4y'' - 16y' - 64y = 0$

$\lambda^2(\lambda+4) - 16(\lambda+4) = 0$

$(\lambda-4)(\lambda+4)(\lambda+4) = 0$

$\lambda = 4, -4, -4$

$$\boxed{y = c_1e^{4x} + c_2e^{-4x} + c_3xe^{-4x}}$$

9. Could a *third* order HLDE have general solution $y = C_1e^{3x} + C_2xe^{3x}$? Explain.

NO. should be 3 L.I. functions in solution.

10. Solve using the Method of Undetermined Coefficients: $y'' + y' - 6y = e^{2x}$.

$y_p = Ae^{2x}$ overlaps with y_h

$\lambda^2 + \lambda - 6 = 0$

$(\lambda+3)(\lambda-2) = 0$

$\lambda = 2, -3$

$y_h = C_1e^{2x} + C_2e^{-3x}$

So $y_p = Axe^{2x}$, $y_p' = Ae^{2x} + 2Axe^{2x}$

$y_p'' = 2Ae^{2x} + 2Ae^{2x} + 4Axe^{2x} = 4Ae^{2x} + 4Axe^{2x}$

plug into DE:

$4Ae^{2x} + 4Axe^{2x} + Ae^{2x} + 2Axe^{2x} - 6Axe^{2x} = e^{2x}$

$5Ae^{2x} = e^{2x}$

$A = \frac{1}{5}$

$\boxed{y = C_1e^{2x} + C_2e^{-3x} + \frac{1}{5}xe^{2x}}$

11. Solve using Variation of Parameters: $y'' + 25y = 3$.

$\lambda^2 + 25 = 0$

$\lambda = \pm 5i$

$y_H = \underset{y_1}{C_1\cos 5x} + \underset{y_2}{C_2\sin 5x}$

$$u_1' = \frac{\begin{vmatrix} 0 & \sin 5x \\ 3 & \cos 5x \end{vmatrix}}{\begin{vmatrix} \cos 5x & \sin 5x \\ -5\sin 5x & 5\cos 5x \end{vmatrix}} = \frac{-3\sin 5x}{5\cos^2 5x + 5\sin^2 5x} = \frac{-3}{5}\sin 5x$$

$\rightarrow u_1 = \frac{3}{25}\cos 5x$

$$u_2' = \frac{\begin{vmatrix} \cos 5x & 0 \\ -5\sin 5x & 3 \end{vmatrix}}{5} = \frac{3}{5}\cos 5x \rightarrow u_2 = \frac{3}{25}\sin 5x$$

$y_p = \frac{3}{25}\cos 5x \cdot \cos 5x + \frac{3}{25}\sin 5x \cdot \sin 5x = \frac{3}{25}$

$\boxed{y = C_1\cos(5x) + C_2\sin(5x) + \frac{3}{25}}$