Math 184 Exam 1 SHOW ALL WORK Name Key

1. Solve each system (if possible). State the solution VERY clearly. You may use a calculator to put each augmented matrix in RREF.

a. $\begin{cases} 2x+3y-3z=2 \\ 3x-y+4z=4 \\ x-5y+z=4 \\ 4x+6y-3z=5 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

no solution

b. $\begin{cases} 2x+3y-3z=2 \\ 3x-y+4z=4 \\ 7x+5y-2z=5 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{9}{11} & 0 \\ 0 & 1 & -\frac{17}{11} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

no solution

8	9 1 5 7 6 7
7	1
6	6 0 4
5	9 8
4	6 7
3	6

2. If possible, find a so that: $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a & 6 \\ 6 & a \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 3 & 3 \end{bmatrix}$. Show your work.

$3a+12=3$
$a=-3$

$18+2a=12$
$a=-3$

$a=-3$

$a+6=3$
$a=3$

$6+a=3$
$a=-3$

3. Find all values of the scalar a for which matrix $\begin{bmatrix} 1 & 2 & 3 \\ a & 5 & 0 \\ 5 & a & 0 \end{bmatrix}$ is *not* invertible.

$\begin{vmatrix} 1 & 2 & 3 \\ a & 5 & 0 \\ 5 & a & 0 \end{vmatrix} = 0 \longrightarrow 3\begin{vmatrix} a & 5 \\ 5 & a \end{vmatrix} = 0$

$3(a^2-25)=0 \rightarrow a=\pm 5$

4. Assume A,B,C,D are nxn matrices, and ~~B~~ A is invertible. Solve the following **matrix** equation for C, using steps appropriate for **matrices**:

ACA + B = D

$ACA = D - B$

$CA = A^{-1}(D-B)$

$C = A^{-1}(D-B)A^{-1}$

5. Let A be an invertible matrix and let k = any nonzero scalar.

Prove that the matrix kA is invertible and that $(kA)^{-1} = \frac{1}{k}A^{-1}$.

$(kA)(\frac{1}{k}A^{-1}) = k(\frac{1}{k})AA^{-1} = 1 \cdot I = I$

$(\frac{1}{k}A^{-1})(kA) = \frac{1}{k} \cdot k A^{-1}A = 1 \cdot I = I$

6. Determine if the set of all 3x1 vectors is a vector space under the standard operation of addition, and with scalar multiplication defined as follows: $k\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2ka \\ 2kb \\ 2kc \end{bmatrix}$. Justify your answer either by showing at least one axiom that fails and how it fails or else by showing the zero and additive inverse vectors.

axiom b fails: $1\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \\ 2c \end{bmatrix} \neq \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

7. Is the set of all lower triangular matrices a *subspace* of M_{nxn}? Justify your answer.

(1) Sum of lower triangular matricies is lower trianguler (Thm 1.12.1)

(2) If A is lower triangular, then so is L•A since L•0=0 so all the zeros will remain all 0's

so yes

8. Let the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be an element of the subspace of M_{2x2} spanned by the set of matrices $\{\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}\}$. What are the conditions necessary for a,b,c,d?

$$\left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & 0 & b \\ 0 & 0 & c \\ 4 & 8 & d \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & 0 & b \\ 0 & 0 & c \\ 0 & 0 & d-4a \end{array}\right]$$

You need: $b = c = 0$ and

$d = 4a$

let $c_1\begin{bmatrix}1&2\\3&4\end{bmatrix}+c_2\begin{bmatrix}5&6\\7&8\end{bmatrix}+c_3\begin{bmatrix}1&2\\1&0\end{bmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

9. Are the matrices $\left\{\begin{bmatrix}1&2\\3&4\end{bmatrix},\begin{bmatrix}5&6\\7&8\end{bmatrix},\begin{bmatrix}3&2\\1&0\end{bmatrix}\right\}$ linearly independent?

Explain, using the definition of linear independence.

$$\left[\begin{array}{ccc|c}1&5&3&0\\2&6&2&0\\3&7&1&0\\4&8&0&0\end{array}\right]\to\left[\begin{array}{ccc|c}1&0&-2&0\\0&1&1&0\\0&0&0&0\\0&0&0&\end{array}\right]$$

since this equation has nontrivial

solutions: $c_1=2c_3$

$c_2=-c_3$

The matrices are <u>not</u> L.I.

10. Can the matrices $\{\begin{bmatrix}1&0\\2&3\end{bmatrix},\begin{bmatrix}1&3\\1&0\end{bmatrix},\begin{bmatrix}0&1\\3&0\end{bmatrix}\}$ form a basis for the set of 2x2

~~symmetric~~ matrices? Explain, using the definition of basis.

No. They <u>are</u> L.I. but do not span $M_{2\times2}$.

$$\left[\begin{array}{ccc|c}1&1&0&a\\0&3&1&b\\2&1&3&c\\3&0&0&d\end{array}\right]\to\left[\begin{array}{ccc|c}1&1&0&a\\0&3&1&b\\0&-1&3&c-2a\\0&-3&0&d-3a\end{array}\right]\to\left[\begin{array}{ccc|c}1&1&0&a\\0&1&\frac{1}{3}&\frac{b}{3}\\0&0&13/3&c-2a+b/3\\0&0&1&d-3a+b\end{array}\right]$$

$$\to\left[\begin{array}{ccc|c}1&1&0&a\\0&1&\frac{1}{3}&b/3\\0&0&1&d-3a+b\\0&0&0&(c-2a+\frac{b}{3})(\frac{-13}{3})+c-2a+b/3\end{array}\right]$$

so matrices where

$\frac{-13}{3}(c-2a+\frac{b}{3})+c-2a+b/3\neq 0$

are not in the span of

the given matrices