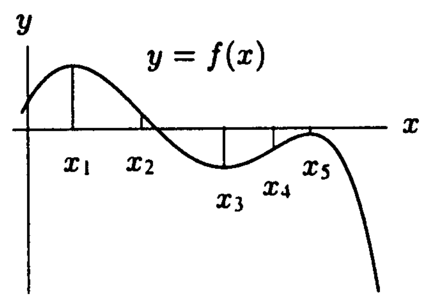
Math 181
Calculus 1
Final Exam
Score__________

Name____________________________________

Read each of the questions carefully and show all your work.

1. The graph of $f(x)$ is given

   (a) Sketch the graph of $f'(x)$ on the same axes.
   (b) Where does $f'(x)$ change its sign?
   (c) Where does $f'(x)$ have a local maximum or minimum?



2. For the function $g(x)$ shown in Figure 2.3, arrange the following numbers in increasing order.

(a) 0 (b) $g'(-2)$ (c) $g'(0)$ (d) $g'(1)$ (e) $g'(3)$

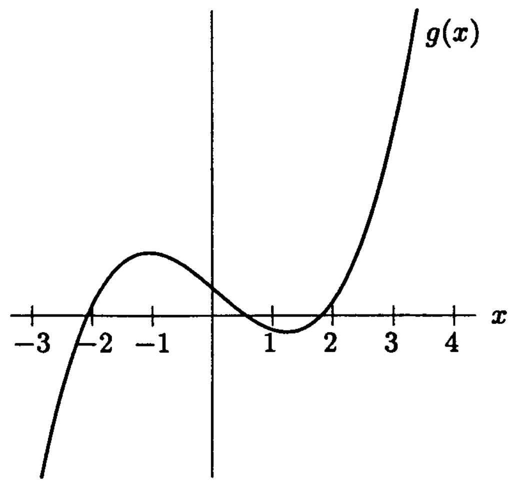


Figure 2.3

3. Given $f(x) = 5x^2 + 3x - 12$,

(a) Find the slope of the tangent line to the curve at $x = -2$.
(b) What is the equation of this tangent line?
(c) Find all points where the curve has a horizontal tangent.

4. For $f(x) = 2x^3 - 9x^2 + 12x + 1, \quad (0 \le x \le 3),$ do the following:

(a) Find f' and f''.

(b) Find the critical points of f.

(c) Find any inflection points.

(d) Evaluate f at the critical points and the endpoints. Identify the global maxima and minima of f.

(e) Sketch f. Indicate clearly where f is increasing or decreasing, and its concavity.

5. Find derivatives for the functions

a. $f(t) = 2te^t - \frac{1}{\sqrt{t}}$ ____________________

b. $w = \frac{5 - 3z}{5 + 3z}$ ____________________

c. $f(t) = \cos^2(3t + 5)$ ____________________

6. Find the indefinite integrals :

(a) $\int (3e^x + 2\sin x)\,dx$

(b) $\int \left(4t + \frac{1}{t}\right)\,dt$

7. Find the exact area of the shaded region in Figure 6.29 between $y = 3x^2 - 3$ and the x-axis.

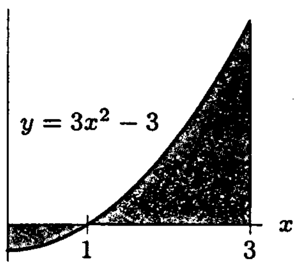
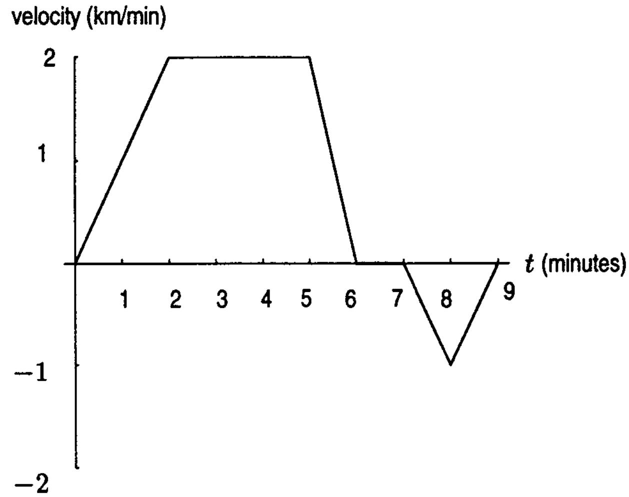


Figure 6.29

8. A car is moving along a straight road from A to B, starting from A at time $t = 0$. Below is the velocity (positive direction is from A to B) plotted against time.



How many kilometers away from A is the car at time $t = 2, 5, 6, 7$, and 9?

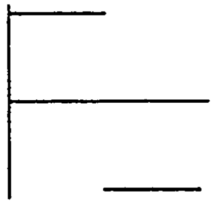
9. Suppose the rate at which ice in a skating pond is melting is given by $\frac{dV}{dt} = 4t + 2$, where V is the volume of the ice in cubic feet, and t is the time in minutes.

(a) Write a definite integral which represents the amount of ice that has melted in the first 4 minutes.
(b) Evaluate the definite integral in part (a).

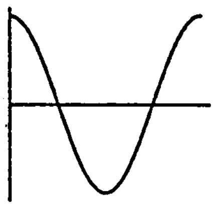
10. Match the following functions with their antiderivatives:

<u>Function</u>

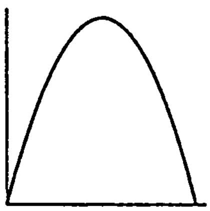
(a)



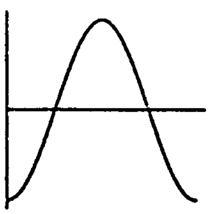
(b)



(c)

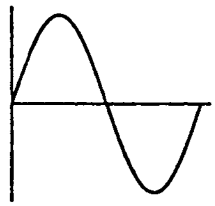


(d)

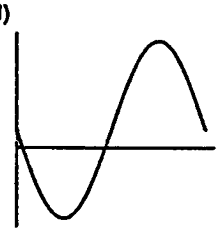


<u>Antiderivative</u>

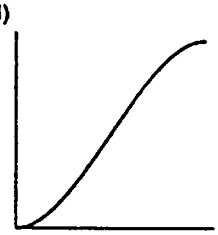
(I)



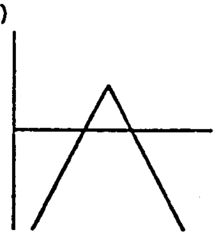
(II)



(III)



(IV)

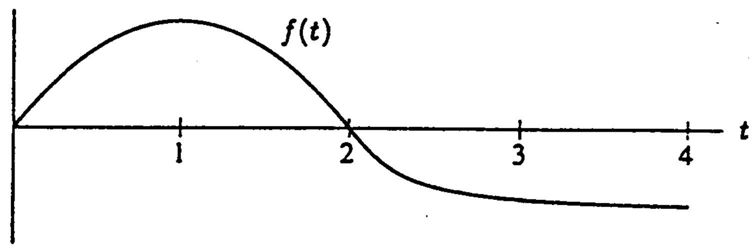


Extra Credit Problem

The function $f(t)$ is graphed below and we define

$$F(x) = \int_0^x f(t)dt.$$

Are the following statements true or false? Give a brief justification of your answer.



(a) $F(x)$ is positive for all x between 2 and 3.

(b) $F(x)$ is decreasing for all x between 1 and 3.

(c) $F(x)$ is concave down for $x = \frac{1}{2}$.

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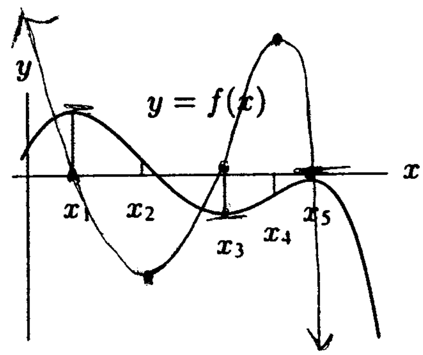
Read each of the questions carefully and show all your work.

1. The graph of $f(x)$ is given

   (a) Sketch the graph of $f'(x)$ on the same axes.

   (b) Where does $f'(x)$ change its sign? x_1, x_3, x_5

   (c) Where does $f'(x)$ have a local maximum or minimum? x_2 : min
   x_4 : max



2. For the function $g(x)$ shown in Figure 2.3, arrange the following numbers in increasing order.

(a) 0 (b) $g'(-2)$ (c) $g'(0)$ (d) $g'(1)$ (e) $g'(3)$

>0 <0 <0 >0

$g'(0)$
$g'(1)$
0
$g'(-2)$
$g'(3)$

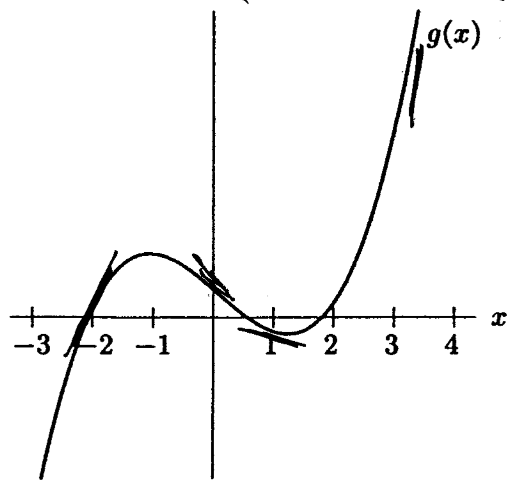


Figure 2.3

3. Given $f(x) = 5x^2 + 3x - 12$,

(a) Find the slope of the tangent line to the curve at $x = -2$.
(b) What is the equation of this tangent line?
(c) Find all points where the curve has a horizontal tangent.

a) $f'(x) = 10x + 3 \rightarrow f'(-2) = 10(-2) + 3 = -17$

b) $f(-2) = 5(-2)^2 + 3(-2) - 12 = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = -17(x - (-2)) \rightarrow y = -17x - 32$

c) $f'(x) = 0 \rightarrow 10x + 3 = 0 \rightarrow x = -\frac{3}{10} = -0.3$

so tangent horizontal when $x = -\frac{3}{10} = -0.3$

$y = f(-\frac{3}{10}) = \frac{-249}{20}$

$= -12.45$

4. For $f(x)=2x^3-9x^2+12x+1, \quad (0 \le x \le 3),$ do the following:

(a) Find f' and f''.

(b) Find the critical points of f.

(c) Find any inflection points.

(d) Evaluate f at the critical points and the endpoints. Identify the global maxima and minima of f.

(e) Sketch f. Indicate clearly where f is increasing or decreasing, and its concavity.

(a) $f'(x) = (3)(2)x^{3-1} - (2)(9)x^{2-1} + (1)(12)x^{1-1} + 0$

$= 6x^2 - 18x + 12$

$f''(x) = (2)(6)x^{2-1} - (1)(18)x^{1-1} + 0$

$= 12x - 18$

(b) The critical points of a function occur when $f'(x) = 0$

$6x^2 - 18x + 12 = 0 \to 6(x^2 - 3x + 2) = 0$

$\to 6(x-2)(x-1) = 0$

$x = 2,\ x = 1$ c.p.'s

$\boxed{\text{Critical Points: } (1, 6) \text{ and } (2, 5)}$

(c) I.P. at $f''(x) = 0 \to 12x - 18 = 0 \to x = \frac{3}{2}$

$\boxed{\text{Inflection Point: } (\tfrac{3}{2}, \tfrac{11}{2}) \text{ or } (1\tfrac{1}{2}, 5\tfrac{1}{2})}$

(d) $f(0) = 1 \to$ global min

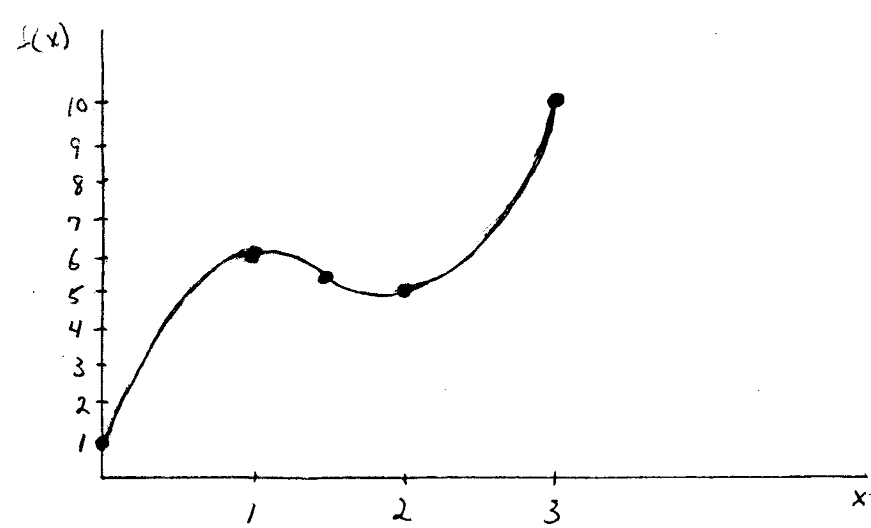
$f(3) = 10 \to$ global max

$f(2) = 5$

$f(1) = 6$

e) $f(3) = 10 \to (3, 10)$

$f(0) = 1 \to (0, 1)$



5. Find derivatives for the functions

a. $f(t) = 2te^t - \frac{1}{\sqrt{t}}$

$f'(t) = 2e^t + 2te^t + \frac{1}{2}t^{-3/2}$

b. $w = \frac{5-3z}{5+3z}$

$w' = \frac{-3(5+3z) - 3(5-3z)}{(5+3z)^2}$

$w' = \frac{-15-9z \;\; -15+9z}{(5+3z)^2}$

$w' = \frac{-30}{(5+3z)^2}$

c. $f(t) = \cos^2(3t+5)$

$f(t) = [\cos(3t+5)]^2$

$f'(t) = 2\cos(3t+5) \cdot -\sin(3t+5) \cdot 3$

$= -6\cos(3t+5)\sin(3t+5)$

6. Find the indefinite integrals :

(a) $\int (3e^x + 2\sin x)\,dx$

(b) $\int \left(4t + \frac{1}{t}\right) dt$

$$\int \left(3(e^x) + 2(\sin x)\right) dx$$

$$= 3(e^x) - 2(-\cos x)$$

$$= 3e^x - 2\cos x + C$$

$$\int \left(4t + \frac{1}{t}\right) dt$$

$$4\left(\frac{1}{1+1}\right)t^{1+1} + \ln|t|$$

$$= \frac{4}{2}t^2 + \ln|t|$$

$$= 2t^2 + \ln|t| + C$$

7. Find the exact area of the shaded region in Figure 6.29 between $y = 3x^2 - 3$ and the x-axis.

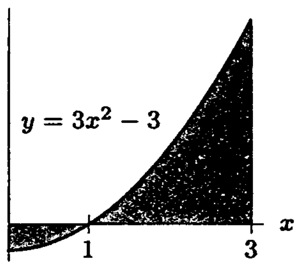


Figure 6.29

$$\int_0^3 |y|\,dx = -\int_0^1 (3x^2 - 3)\,dx + \int_1^3 (3x^2 - 3)\,dx$$

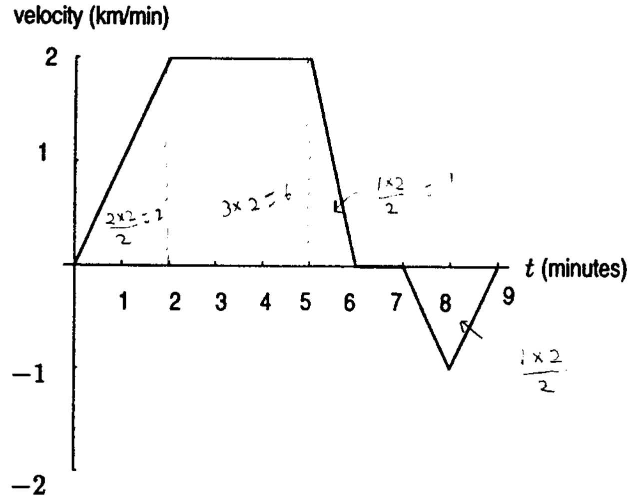
$$= -\left[x^3 - 3x\right]_0^1 + \left[x^3 - 3x\right]_1^3$$

$$= -(1 - 3) + (27 - 9) - (1 - 3)$$

$$= 2 + 18 + 2$$

$$= 22$$

8. A car is moving along a straight road from A to B, starting from A at time $t = 0$. Below is the velocity (positive direction is from A to B) plotted against time.



How many kilometers away from A is the car at time $t = 2, 5, 6, 7$, and 9?

t	2	5	6	7	9
Km away from A	2	8	9	9	8

9. Suppose the rate at which ice in a skating pond is melting is given by $\frac{dV}{dt} = 4t + 2$, where V is the volume of the ice in cubic feet, and t is the time in minutes.

(a) Write a definite integral which represents the amount of ice that has melted in the first 4 minutes.
(b) Evaluate the definite integral in part (a).

a) $dV = (4t + 2)\,dt$

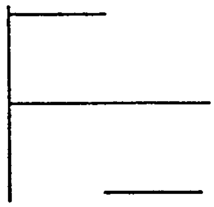
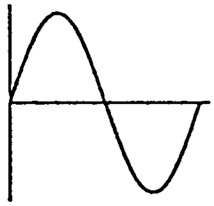
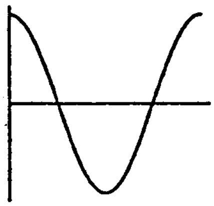
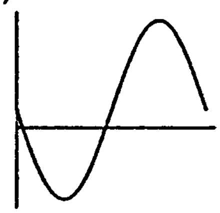
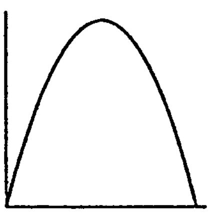
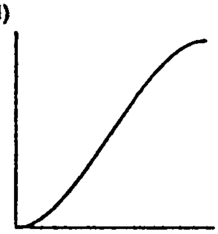
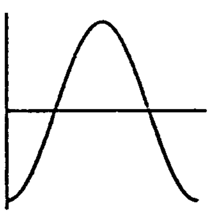
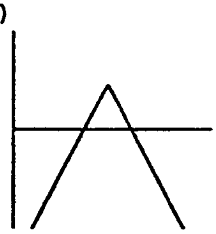
$V = \int_0^4 (4t+2)\,dt$

b) $V = 2t^2 + 2t \Big]_0^4$

$= 32 + 8 - 0$

$V = 40 \text{ ft}^3$ of ice melted in first 4 minutes

10. Match the following functions with their antiderivatives:

<u>Function</u>	<u>Antiderivative</u>
IV (a) 	(I) 
I (b) 	(II) 
III (c) 	(III) 
II (d) 	(IV) 

Extra Credit Problem

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$$F(x) = \int_0^x f(t)dt.$$

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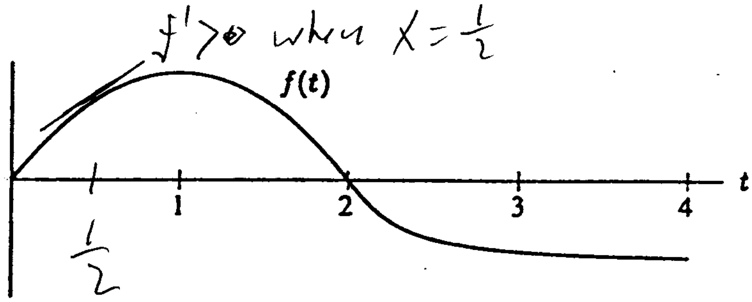

(a) $F(x)$ is positive for all x between 2 and 3.

(b) $F(x)$ is decreasing for all x between 1 and 3.

(c) $F(x)$ is concave down for $x = \frac{1}{2}$.

(a) T: Area above x-axis on $[0,2]$ exceeds area below x-axis on $[2,3]$

(b) F: $F(x)$ is increasing on $[0,2]$ since area is increasing

(c) F: $F''(x) = f'(x)$: $F''(\frac{1}{2}) = f'(\frac{1}{2}) > 0$: concave up