Math 181

Final Exam Name______________________________

For full credit, please show your work or explain your answer for each problem.

1. Find the following limits.

   a. $\lim_{x \to 0} \frac{\sin 3x}{x}$

   b. $\lim_{x \to \infty} \frac{x^2 - 3}{2x^2 + 4x}$

2. Find the derivatives of the following functions.

   a. $f(x) = e^{-x} \sin 3x$

   b. $f(x) = 3\ln(\cos x) + \arctan(2x^2)$

3. Find the equation of the line tangent to the curve $x^2y-3y^2=2x$ at the point $(-1,1)$.

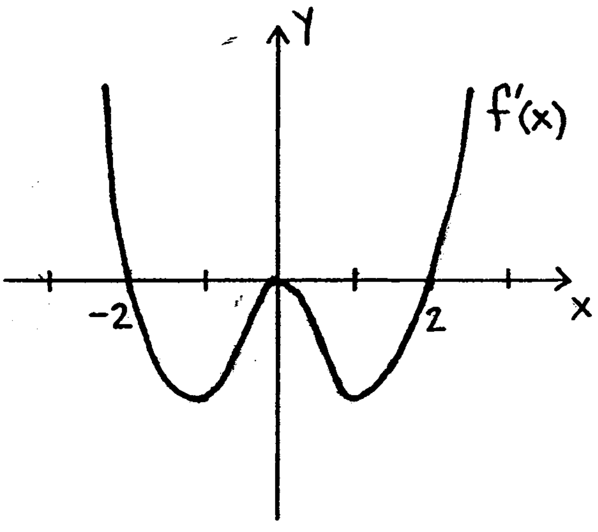
4. Given the graph of the derivative $f'(x)$, find the values of x where the function $f(x)$ has

a. critical points.

b. local maxima.

c. local minima.

d. inflection points.



5. Given $f(x) = x^3 + 3x^2 + 2$.

a. Find the intervals where f is increasing and decreasing, and find the local maxima and minima.

b. Find the intervals where f is concave up and concave down, and find the inflection points.

6. For the function $g(x)$ shown in Figure 2.3, arrange the following numbers in increasing order.

(a) 0 (b) $g'(-2)$ (c) $g'(0)$ (d) $g'(1)$ (e) $g'(3)$

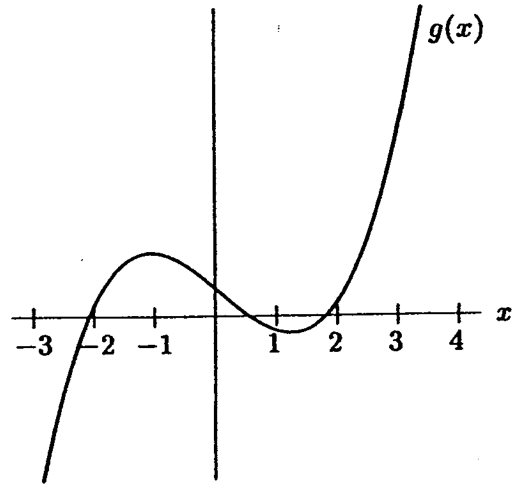


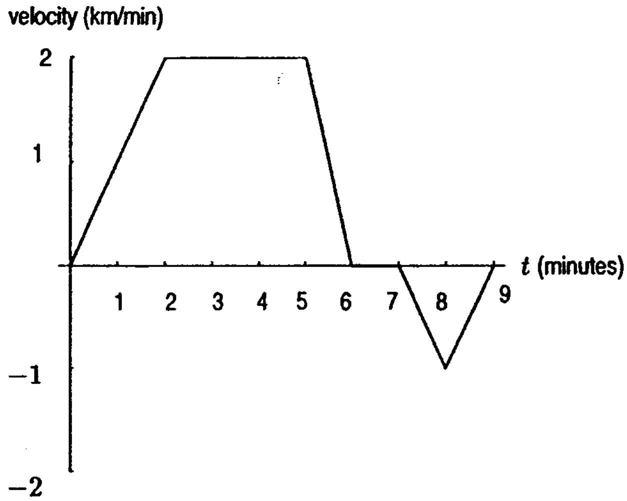
Figure 2.3

7. Find the indefinite integrals.

a. $\int\left(3\sqrt{x}+\sin x\right)dx$

b. $\int\frac{x^2+2x+1}{x}dx$

8. A car is moving along a straight road from A to B, starting from A at time $t = 0$. Below is the velocity (positive direction is from A to B) plotted against time.



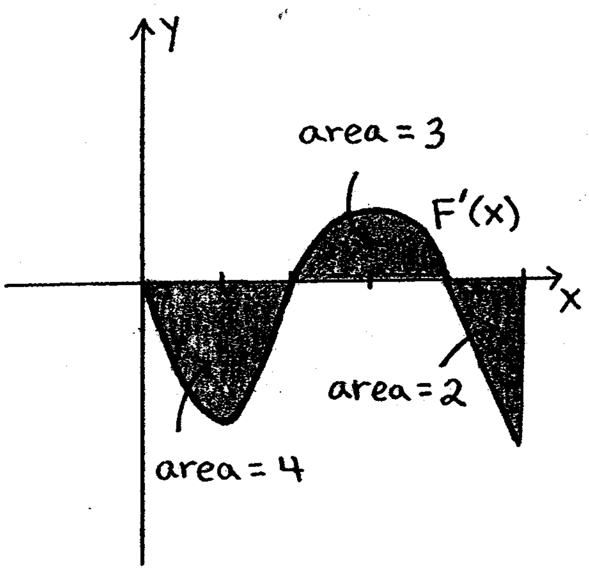
How many kilometers away from A is the car at time $t = 2, 5, 6, 7$, and 9?

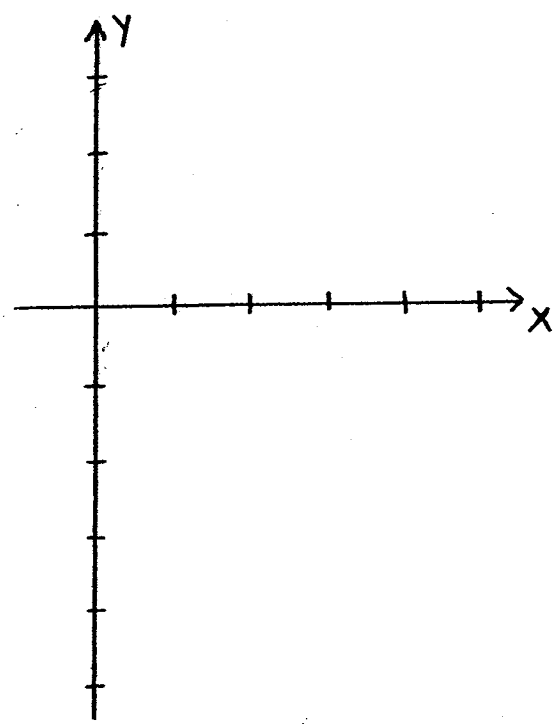
9. Suppose the rate at which ice in a skating pond is melting is given by $\dfrac{dV}{dt} = 4t + 2$, where V is the volume of the ice in cubic feet, and t is the time in minutes.

(a) Write a definite integral which represents the amount of ice that has melted in the first 4 minutes.
(b) Evaluate the definite integral in part (a).

10. Sketch and find the area between the graph of $f(x) = x(x+2)(x-3)$ and the x-axis in the interval $[-2,3]$.

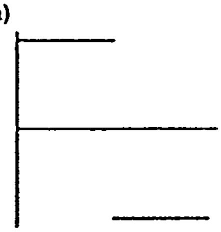
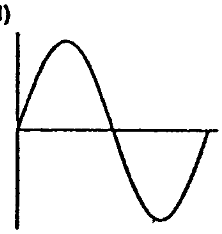
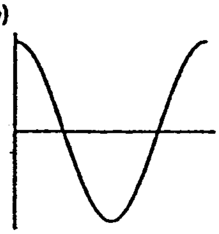
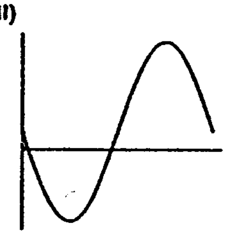
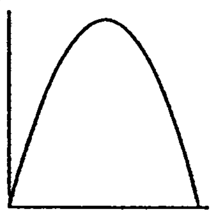
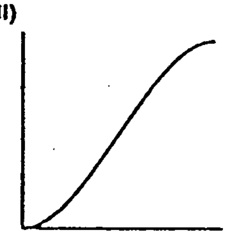
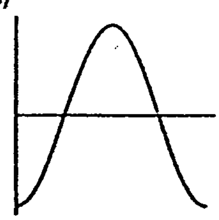
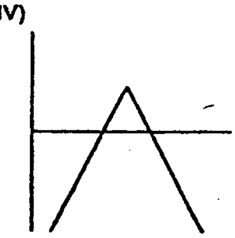
11. Given the graph of the derivative $F'(x)$ with the areas as shown in the graph below, and $F(0) = 0$, complete the table and then sketch the graph of the function $F(x)$. Show clearly where the graph is increasing, decreasing, concave up, and concave down.





x	0	2	4	5
F(x)	0			

12. Match the following functions with their antiderivatives:

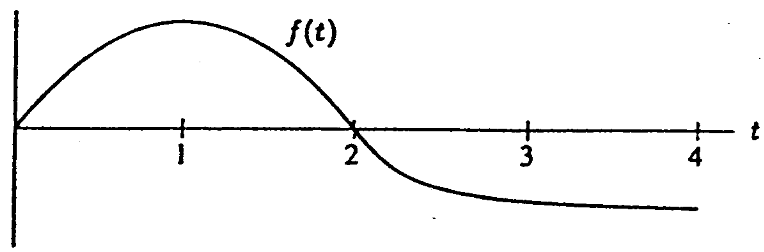
Function	Antiderivative
(a) 	(I) 
(b) 	(II) 
(c) 	(III) 
(d) 	(IV) 

Extra Credit Problem

The function $f(t)$ is graphed below and we define

$$F(x) = \int_0^x f(t)dt.$$

Are the following statements true or false? Give a brief justification of your answer.



(a) $F(x)$ is positive for all x between 2 and 3.

(b) $F(x)$ is decreasing for all x between 1 and 3.

(c) $F(x)$ is concave down for $x = \frac{1}{2}$.

Total: 126 + 14 Extra Credit

Math 181

Final Exam Name________________

For full credit, please show your work or explain your answer for each problem.

10

1. Find the following limits.

a. $\lim_{x\to 0}\frac{\sin 3x}{x}$ $= \lim_{x\to 0} \frac{3\cos 3x}{1} = \boxed{3}$

b. $\lim_{x\to\infty}\frac{x^2-3}{2x^2+4x}$ $= \lim_{x\to\infty} \frac{1-\frac{3}{x^2}}{2+\frac{4}{x}} = \boxed{\frac{1}{2}}$

10

2. Find the derivatives of the following functions.

a. $f(x)=e^{-x}\sin 3x$

$f'(x) = -e^{-x}\sin 3x + 3e^{-x}\cos 3x$

$= \boxed{e^{-x}(3\cos 3x - \sin 3x)}$

b. $f(x)=3\ln(\cos x)+\arctan(2x^2)$

$f'(x) = 3\cdot\frac{1}{\cos x}\cdot(-\sin x) + \frac{1}{1+4x^4}\cdot 4x$

$= \boxed{-3\tan x + \frac{4x}{1+4x^4}}$

15

3. Find the equation of the line tangent to the curve $x^2y - 3y^2 = 2x$ at the point $(-1,1)$.

$$\frac{d}{dx}(x^2y) - \frac{d}{dx}(3y^2) = \frac{d}{dx}(2x)$$

$$2xy + x^2\frac{dy}{dx} - 6y\frac{dy}{dx} = 2$$

$$(x^2 - 6y)\frac{dy}{dx} = 2 - 2xy$$

$$\frac{dy}{dx} = \frac{2-2xy}{x^2-6y}$$

$$m = \left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{2+2}{1-6} = -\frac{4}{5}$$

$$y - 1 = -\frac{4}{5}(x+1)$$

$$y = -\frac{4}{5}x - \frac{4}{5} + 1$$

$$\boxed{y = -\frac{4}{5}x + \frac{1}{5}}$$

10

4. Given the graph of the derivative $f'(x)$, find the values of x where the function $f(x)$ has

a. critical points.

$-2, 0, 2$

b. local maxima.

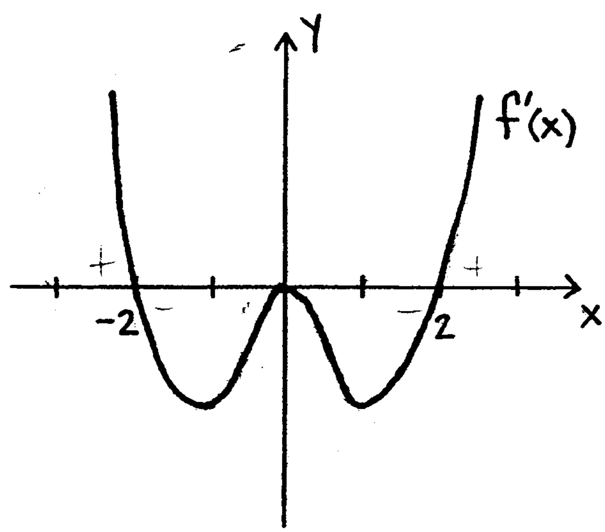
-2

c. local minima.

2

d. inflection points.

$-1, 0, 1$



15

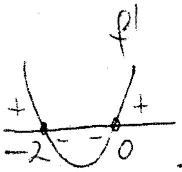
5. Given $f(x) = x^3 + 3x^2 + 2$.

a. Find the intervals where f is increasing and decreasing, and find the local maxima and minima.

$$f'(x) = 3x^2 + 6x$$

$$f'(x) = 0 \implies 3x(x+2) = 0$$

critical points : $-2, 0$



Since $f'(x) > 0$ on the intervals $(-\infty, -2)$, $(0, \infty)$,
f is increasing on $(-\infty, -2)$, $(0, \infty)$.

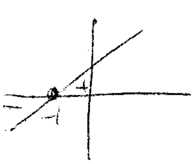
Since $f'(x) < 0$ on the interval $(-2, 0)$,
f is decreasing on $(-2, 0)$.

By the first derivative test there is a local maximum at $x = -2$ and a local minimum at $x = 0$.

b. Find the intervals where f is concave up and concave down, and find the inflection points.

$$f''(x) = 6x + 6$$

$$f''(x) = 0 \implies x = -1$$



$f''(x) > 0$ on $(-1, \infty)$ $\implies$ f is concave up on $(-1, \infty)$

$f''(x) < 0$ on $(-\infty, -1)$ $\implies$ f is concave down on $(-\infty, -1)$

Since f'' changes its sign at $x = -1$, there is an inflection point at $x = -1$.

8

6. For the function $g(x)$ shown in Figure 2.3, arrange the following numbers in increasing order.

(a) 0 (b) $g'(-2)$ (c) $g'(0)$ (d) $g'(1)$ (e) $g'(3)$

$+$ $-$ $-$ $+$

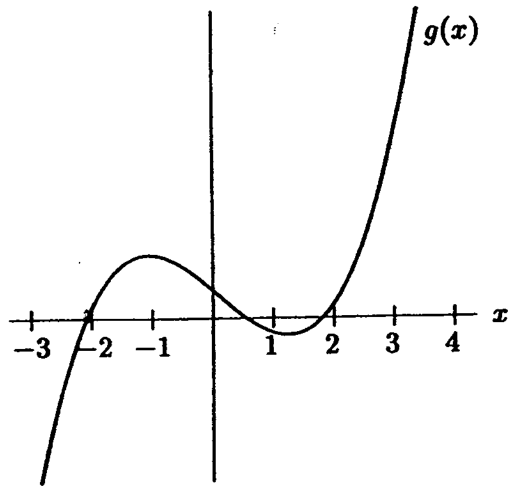


Figure 2.3

$g'(0) < g'(1) < 0 < g'(-2) < g'(3)$

c, d, a, b, e

10

7. Find the indefinite integrals.

a. $\int \left(3\sqrt{x} + \sin x\right) dx$ $= 3\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - \cos x + C$

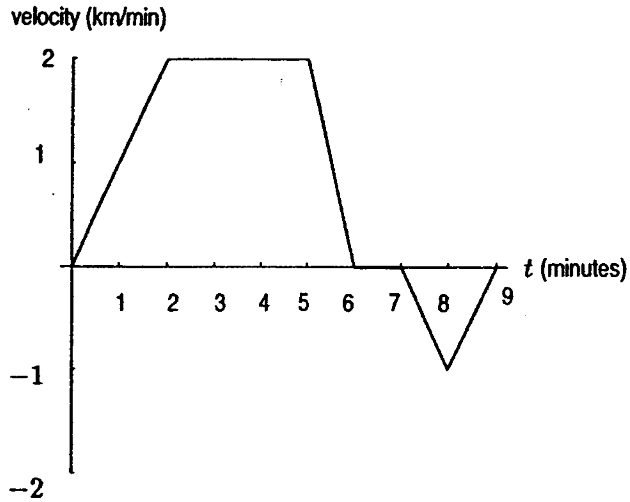
$= \boxed{2x^{\frac{3}{2}} - \cos x + C}$

b. $\int \dfrac{x^2 + 2x + 1}{x} dx$ $= \int \left(x + 2 + \frac{1}{x}\right) dx =$

$\boxed{\dfrac{x^2}{2} + 2x + \ln|x| + C}$

8. A car is moving along a straight road from A to B, starting from A at time $t = 0$. Below is the velocity (positive direction is from A to B) plotted against time.

8



How many kilometers away from A is the car at time $t = 2, 5, 6, 7,$ and 9?

t	2	5	6	7	9
d	2	8	9	9	8

9. Suppose the rate at which ice in a skating pond is melting is given by $\frac{dV}{dt} = 4t + 2$, where V is the volume of the ice in cubic feet, and t is the time in minutes.

8+2

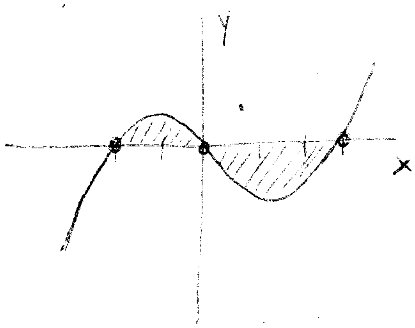
(a) Write a definite integral which represents the amount of ice that has melted in the first 4 minutes.
(b) Evaluate the definite integral in part (a).

(a) $\int_0^4 (4t+2)\,dt$

(b) $\int_0^4 (4t+2)\,dt = 2t^2 + 2t \Big|_0^4 = 2\cdot 16 + 8 = \boxed{40 \text{ ft}^3}$

12 + 3

10. Sketch and find the area between the graph of $f(x) = x(x+2)(x-3)$ and the x-axis in the interval $[-2,3]$.



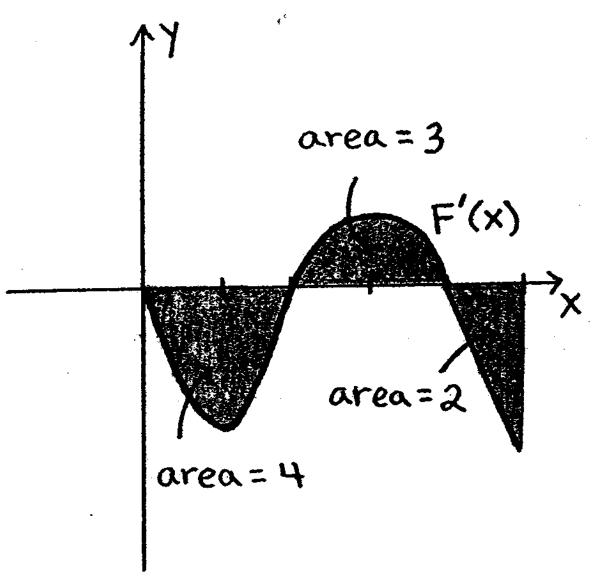
$$A = \int_{-2}^{c} x(x+2)(x-3)\,dx - \int_{0}^{3} x(x+2)(x-3)\,dx =$$

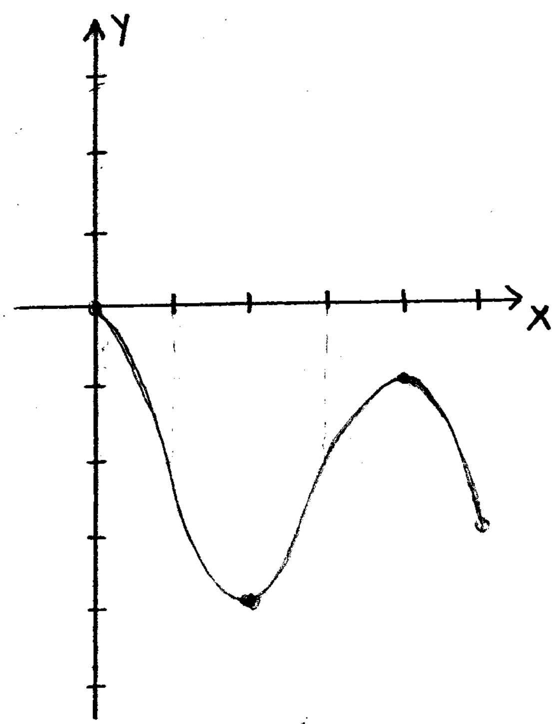
$$5\tfrac{1}{3} + 15\tfrac{3}{4} = \boxed{21\tfrac{1}{12}}$$

$$\frac{253}{12}$$

12

11. Given the graph of the derivative $F'(x)$ with the areas as shown in the graph below, and $F(0) = 0$, complete the table and then sketch the graph of the function $F(x)$. Show clearly where the graph is increasing, decreasing, concave up, and concave down.





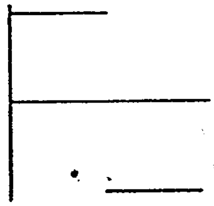
X	0	2	4	5
F(x)	0	-4	-1	-3

8

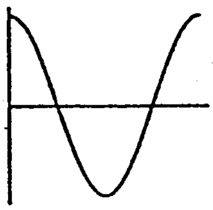
12. Match the following functions with their antiderivatives:

Function

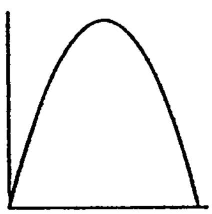
(a)



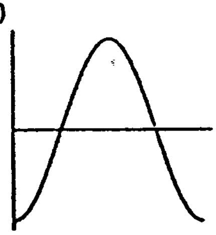
(b)



(c)

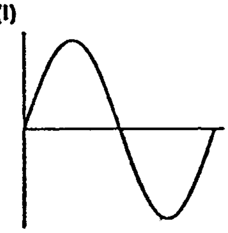


(d)

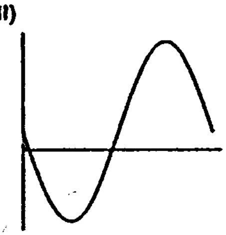


Antiderivative

(I)



(II)



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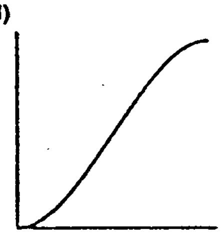
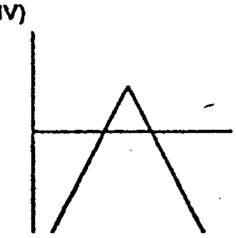


(IV)



A IV

B I

C III

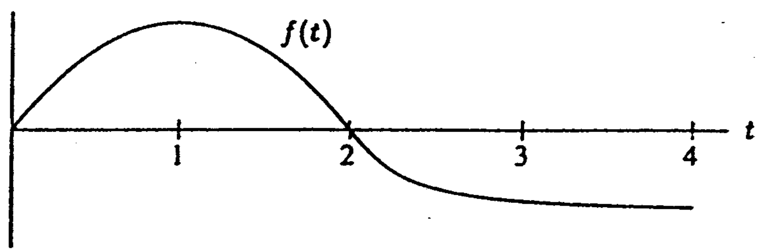
D II

Extra Credit Problem

The function $f(t)$ is graphed below and we define

$$F(x) = \int_0^x f(t)dt.$$

Are the following statements true or false? Give a brief justification of your answer.



(a) $F(x)$ is positive for all x between 2 and 3. TRUE

(b) $F(x)$ is decreasing for all x between 1 and 3. FALSE

(c) $F(x)$ is concave down for $x = \frac{1}{2}$. FALSE

(a) The area above the t-axis from 0 to 2 is greater than the area below the t-axis from 2 to 3.

(b) F is still increasing on the interval $(1,2)$.

(c) At $t = \frac{1}{2}$ the derivative (of f) is positive, thus F is concave up.

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