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taking ...

Illowsky – Chapt. 4, 6, 7, & 8
Larson – Chapt. 4, 5, & 6

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Math 123 Exam 2 SHOW ALL WORK Name

M.t.dm

1. The given table shows how many employees different small businesses from a random sample employ. Let X = the number of employees per business. Build a probability distribution table for X, then find μ, the expected number of employees per business.

Number of Employees	1	2	3	4	5
	(3)	(30)	(147)	(124)	(15) (319)
Number of Businesses	3	15	49	31	3/101

(3.158 × 101 = 318.9

X	P(X)	X · P(X)
1	3/101 = 0.02970	0.02970
2	15/101 = 0.14851	0.29702
3	49/101 = 0.48514	1.45544
4	31/101 = 0.30693	1.22772
5	3/101 = 0.02970	0.14851

(expected # of employees per business)
$\mu = \boxed{3.15841}$

2. Suppose 20% of all small businesses have only one employee. You choose 6 small businesses at random. Justify your answers to the following:

p=0.20, n

a. How many of the businesses in your sample would you *expect* to have one employee?

$6 \times 0.20 = \boxed{1.2}$

b. What is the probability that *exactly 4* of the businesses have one employee?

$P(X=4) = \boxed{0.01536}$

binompdf (6, 0.20, 4)

c. What is the probability that *at least 2* of the businesses have one employee?

$P(X \geq 2) = \boxed{0.34464}$

binomcdf (6, 0.20, 1) = 0.65536

1 − 0.65536 = 0.34464

mean

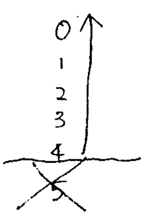
3. If the mean number of employees per small business is 2.8, what is the probability that a randomly chosen small business will have 2 employees?

$P(X=2)$

possionpdf $(2.8, 2) = \boxed{0.2383}7$

$p=0.20$

4. Suppose 20% of all small businesses have exactly one employee. If you repeatedly ask small business owners how many employees they have, what is the probability that you will need to ask *no more than 4* owners before finding one that has only one employee?

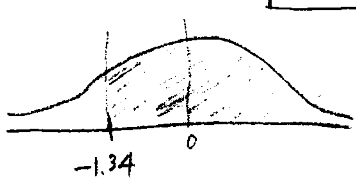


$P(X<4)$

geometcdf $(0.20, 4) = \boxed{0.5904}$

5. Use the standard normal distribution table in the text (or a calculator) to find the following probabilities:

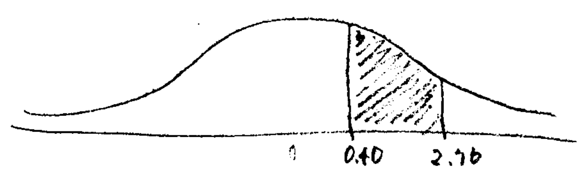
a. $P(Z > -1.34) = \boxed{0.9099}$



normalcdf $(-1.34, 1{,}000) = 0.90987$

b. $P(0.40 < Z < 2.76) = \boxed{0.3417}$

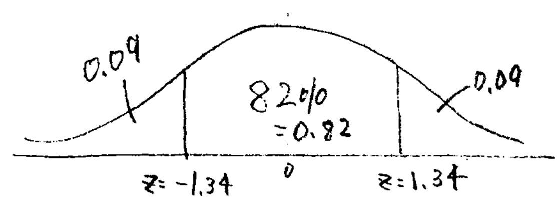
normalcdf $(0.40, 2.76) = 0.34168$



6. Find the value of Z such that 82% of the area under the standard normal curve lies between –Z and Z.

$\boxed{(-1.34 < Z < 1.34)}$

~~normalcdf (−1.34, 1.34) = 0.819754~~

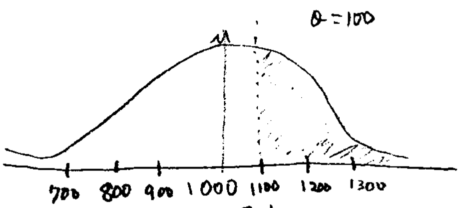


invNorm $(0.09) = -1.34$

$Z = \frac{X - \mu}{\sigma}$

7. Suppose that scores on an exam are normally distributed with a mean of $\mu = 1000$ and a standard deviation of $\sigma = 100$. Show your work on the following:

a. What is the probability that a randomly chosen exam score will exceed 1100?



$P(X > 1100)$

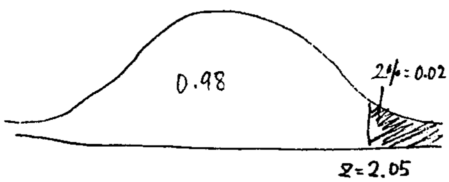
$Z = \frac{1100 - 1000}{100} = \frac{100}{100} = 1$

$P(Z > 1) = \text{normalcdf}(1, 1{,}000) = \boxed{0.158655}$ (6)

$X = \mu + Z\sigma$

b. What score would be the cutoff for the top 2% of scores?

$Z = \text{invNorm}(0.98) = 2.0537$



$X = 1{,}000 + (2.05)(100)$

$= \boxed{1205}$ scores

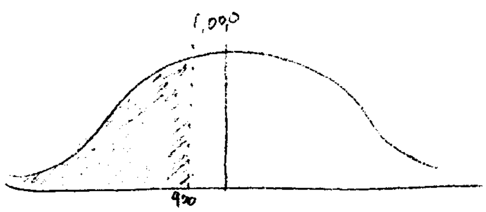
c. If you repeatedly take random samples of size n = 15 exams, and compute the sample mean score for each sample, what values would you expect to find for the mean and standard deviation of the sampling distribution of the sample mean?

$\boxed{\mu_{\bar{x}} = 1{,}000}$ $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{15}} = 25.82\cancel{1988}}$

d. What is the probability that the mean score from a random sample of 30 exams will be less than 970?

n = 30

$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$



$Z = \frac{970 - 1{,}000}{\frac{100}{\sqrt{30}}} = \frac{-30}{18.257} = -1.64\cancel{31}$

$\text{normalcdf}(-1{,}000, -1.64) = \boxed{0.0505}$

8. You wish to estimate the proportion (p) of small businesses that have one employee to within a margin of error of 2 percentage points, with 90% confidence. What is the minimum sample size required, assuming you have no preliminary sample data?

$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2$

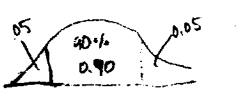
E = 2% points ⇒ 0.02

$\hat{p} = 0.5$

$\hat{q} = 0.5$

c = 90% ⇒ 0.90

$z_c = 1.645$



invNorm(0.05) = 1.645

$n = (0.5)(0.5)\left(\frac{1.645}{0.02}\right)^2$

$= 0.25\ (6765.0625)$

$= 1691.265625 \implies \boxed{1{,}692}$ small businesses

$n=200 \quad n \geq 30$

9. Suppose a survey of 200 randomly chosen businesses found that 160 offer their employees 2 weeks of paid vacation per year. Build a 95% confidence interval for the population proportion of businesses that offer 2 weeks of paid vacation per year.

$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hat{p} - E < P < \hat{p} + E$

$x = 160 \quad n = 200 \quad \hat{p} = \frac{160}{200} = 0.8 \quad \hat{q} = 0.2 \quad c = 95\% \to 0.95 \quad z_c = 1.960$

$E = 1.960\sqrt{\frac{(0.8)(0.2)}{200}} = 0.05543$

$0.8 - 0.0554 < P < 0.8 + 0.0554$

$\boxed{0.7446 < P < 0.8554}$

$n=34$

10. Suppose a random sample of 34 businesses found that the mean number of employees was 23.7 with standard deviation 3.5. Build a 99% confidence interval for the population mean number of employees per business.

$n = 34$

$\bar{x} = 23.7$

$s = 3.5$

σ not known

$c = 99\% = 0.99$

$E = t_c\left(\frac{s}{\sqrt{n}}\right)$

$t_c = 2.733$ (from chart)

$E = (2.733)\left(\frac{3.5}{\sqrt{34}}\right)$

$E = 1.64$

Confidence interval:

$23.7 - 1.64 < \mu < 23.7 + 1.64$

$\boxed{22.06 < \mu < 25.34}$

11. Suppose a random sample of the hourly wage for n = 5 employees at a business yielded the following: \$19.32, \$12.50, \$11.12, \$12.00, \$9.56. Assuming that hourly wages are normally distributed, build a 95% confidence interval for the population mean hourly wage for employees at this business.

$E = t_c\left(\frac{s}{\sqrt{n}}\right)$

$\bar{x} - E < \mu < \bar{x} + E$

because σ not known

$\bar{x} = 12.9 \quad s = 3.7\cancel{586} \quad c = 95\% \Rightarrow 0.95 \quad n = 5 \quad df = 4 \quad t_c = 2.776$

$E = 2.776\left(\frac{3.76}{\sqrt{5}}\right) = 4.6679$

$12.9 - 4.6679 < \mu < 12.9 + 4.6679$

$\boxed{8.2321 < \mu < 17.5679}$

-2

needed since $n < 30$

12. Discuss the importance of the normal distribution of wages in Problem 11.

$5 < 30$ but ~~I still can build a CI because I know that Standard deviation~~.

~~Also~~, #11 told me that "assuming that hourly wages are normally distributed"