Differentiation Formulas

1. $\big(f(x) \pm g(x)\big)' = f'(x) \pm g'(x)$
2. $\big(kf(x)\big)' = kf'(x)$
3. $\big(f(x)g(x)\big)' = f(x)g'(x) + g(x)f'(x)$
4. $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
5. $\Big(f\big(g(x)\big)\Big)' = f'\big(g(x)\big) \cdot g'(x)$
6. $\frac{d}{dx}\,(x^n) = nx^{n-1}$
7. $\frac{d}{dx}(a^x) = a^x \ln a \quad (a > 0)$
8. $\frac{d}{dx}\,(e^x) = e^x$
9. $\frac{d}{dx}\,\big(e^{u(x)}\big) = \frac{du}{dx}e^{u(x)}$
10. $\frac{d}{dx}\,(\ln x) = \frac{1}{x}$
11. $\frac{d}{dx}(\sin x) = \cos x$
12. $\frac{d}{dx}(\cos x) = -\sin x$
13. $\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}$

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\,dx = \frac{1}{n+1}\,x^{n+1} + c, \;\; n \neq -1$
2. $\int \frac{1}{x}\,dx = \ln|x| + c$
3. $\int a^x\,dx = \frac{1}{\ln a}\,a^x + c\,, \;\; a > 0$
4. $\int \ln x\;dx = x\,\ln x - x + c$
5. $\int \sin x\,dx = -\cos x + c$
6. $\int \cos x\,dx = \sin x + c$
7. $\int \tan x\,dx = -\ln|\cos x| + c$

II. Products of $e^x, cos\,x, and\ sin\,x$

8. $\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}[a\sin(bx) - b\cos(bx)] + c$
9. $\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}[a\cos(bx) + b\sin(bx)] + c$
10. $\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}\,[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + c\,, \;\; a \neq b$
11. $\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}\,[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + c\,, \;\; a \neq b$
12. $\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}\,[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + c\,, \;\; a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x, e^x, cos\,x, sin\,x$

13. $\int x^n \ln x\;dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + c \qquad n \neq -1$
14. $\int p(x)e^{ax}dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}dx$

$= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \;\ldots\ldots$ **(+ - + -……→signs alternate)**

15. $\int p(x)\sin ax\,dx = -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\;dx$

$= -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots$

(- ++ - - + +...) (Signs alternate in pairs after first term)

16. $\int p(x)\cos ax\,dx = \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\,dx$

$= \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots$

(+ + - - + + - -...) (Signs alternate in pairs)

IV. Integer Powers of $\boldsymbol{sin\,x\ and\ cos\,x}$

17. $\int sin^n x\,dx = -\frac{1}{n}sin^{n-1}x\cos x + \frac{n-1}{n}\int sin^{n-2}x\,dx$ *when n is positive*
18. $\int cos^n x\,dx = \frac{1}{n}cos^{n-1}x\sin x + \frac{n-1}{n}\int cos^{n-2}x\,dx$ *when n is positive*
19. $\int \frac{1}{sin^m x}dx = \frac{-1}{m-1}\cdot\frac{\cos x}{sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{sin^{m-2}x}dx$ *when $m \neq 1$ and m is positive*
20. $\int \frac{1}{\sin x}dx = \frac{1}{2}\ln\left|\frac{(\cos x)-1}{(\cos x)+1}\right| + c$
21. $\int \frac{1}{cos^m x}dx = \frac{1}{m-1}\cdot\frac{\sin x}{cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{cos^{m-2}x}dx$ *when $m \neq 1$ and m is positive*
22. $\int \frac{1}{\cos x}dx = \frac{1}{2}\ln\left|\frac{(\sin x)+1}{(\sin x)-1}\right| + c$
23. $\int sin^m x\,cos^n x\,dx$:

If *m* is odd, let $w = \cos x$. If *n* is odd, let $w = \sin x$. If both *m* and *n* are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $sin^2 x + cos^2 x = 1$), and use IV-17 or IV-18. If *m* and *n* are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. If both *m* and *n* are even and negative, the substitution $w = \cos x$ converts the integral into a rational function which can be integrated by the method of partial fractions.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}arctan\frac{x}{a} + c, \quad a \neq 0$
25. $\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}arctan\frac{x}{a} + c, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + c, \quad a \neq b$
27. $\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + c, \quad a \neq b$

VI. Integrands Involving $\boldsymbol{\sqrt{a^2+x^2}, \sqrt{a^2-x^2}, \sqrt{x^2-a^2}}$, $\boldsymbol{a > 0}$

28. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = arcsin\frac{x}{a} + c$
29. $\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + c$
30. $\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2\pm x^2}}\,dx) + c$
31. $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2-a^2}}\,dx) + c$